Optimization Study of Space Radiators

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A method has been developed for optimizing a rectangular fin with a variable root temperature along the tube length Calculations have been performed for a typical set of system conditions The results indicate that the magnitude of the optimum fin thickness is, for the representative case considered, too small to be structuı ally feasible Hence, the fin thickness, now taken as a constant, should be selected on a minimum thickness basis requiı ed to provide structural integrity Thus, only the fin length would be optimized A comparison was made between the method that treats an optimum constant-length fin and the present analysis for the variable-length fin The results show that the radiator system with the constant-length fin is 1 5% heavieı than the sy stem that uses the vaı iable-length fin Although this represents a small weight increase and therefore justifies the use of a constant-length fin, it has been shown that, in general, large deviations from the optimum design condition would introduce relatively small weight penalties As a result, the 1 5% diffeı ence between the two methods may, in fact, ı epresent a large deviation from the optimum This indicates that strict adherence to optimum design conditions is not geneı ally necessaı y

Nomenclature

A = tube flow area
c_p = specific heat at constant pressure
D = tube diameter
F = factor defined by Eq (9)
h = heat transfer coefficient
k = thermal conductivity
l = fin length
m = mass rate of flow
M = Mach number
$\mathfrak{M}$ = molecular weight
Nu = Nusselt number
P = total pressure
Pr = Prandtl number
Re = Reynolds number
$\mathfrak{R}$ = universal gas constant
Q = heat-transfer rate
T = temperature
t = thickness
U = modified coefficient of heat transmission
V = volume
x = coordinate axis along the fin length
y = coordinate axis along the tube length
$\beta_1, \beta_2, \beta_3$ = parameters defined by Eqs (5a) and (7a)
γ = isentropic exponent
ϵ = hemispherical emissivity
η = transformed coordinate along the y axis
θ = transformed temperature
λ = transformed thickness
μ = absolute viscosity
ξ = transformed coordinate along the x axis
σ = Stefan Boltzmann constant
τ = transformation defined as $\theta(\xi,\eta)/\theta(\xi_l,\eta)$
ψ = volume parameter
Ω = heat-transfer parameter

Subscripts

b = fluid
f = fin
t = tube

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Introduction

POWERPLANTS and cooling systems of vehicles designed to operate in space for long periods of time require heat-rejection systems that are capable of dissipating large quantities of waste heat One method by which this can be accomplished is through the use of an expendable coolant, which would use its heat capacity to absorb waste heat This may be accomplished by regenerative means, i e , by circulation of the coolant in flow passages that are subjected to a heat flux from a source of heat generation such as a powerplant or electronic equipment The hot coolant emerging from the flow passages is then ejected overboard Although feasible for short periods of time, this method of heat rejection is impractical in sustained space operation because of the large weight penalty associated with the coolant requirements

An alternate means of heat rejection would use the same type of system for heat absorption However, instead of ejecting the hot coolant overboard, an additional system would be provided which would allow the hot coolant to reject heat and thus be reusable for cooling purposes Such a system, requiring a fixed amount of coolant for continuous operation, would absorb heat in one part of the system and reject it in another

An effective method for such heat rejection during space operation is surface radiation In this mode of waste heat dissipation, the vehicle skin or auxiliary surfaces are used as radiating surfaces Integral to both constructions is the circulation of a hot coolant through a series of passages that can be built into the skin of the vehicle or to externally provided radiation surfaces In either case, the attached passages are spaced on the radiating surface in such a way as to provide optimum operation, i e , maximum heat rejection with minimum system weight The resulting heat-rejection system is then of the fin-duct type of configuration where heat is dissipated by radiation from both the exposed surface of the duct as well as by conduction along the fin and subsequent radiation from the fin surface

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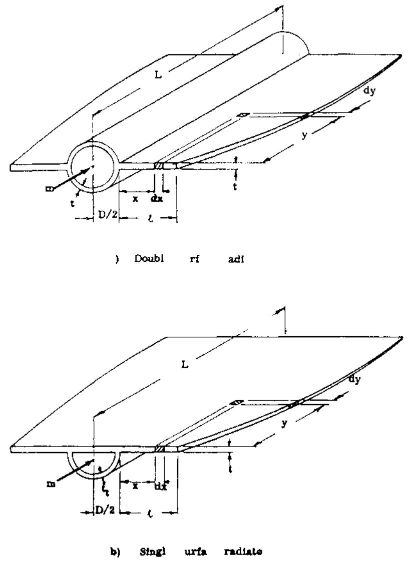

Fig 1 Fin-tube configuration for space-radiator system

Although attractive from the standpoint of weight savings, the integral vehicle skin-radiator system has the disadvantage of being a single-surface radiator In addition, the insulation that may be required to protect the internal structure of the vehicle from the hot coolant may more than offset the weight savings realized by integrating the heat-rejection system with the vehicle skin

The radiator system external to the vehicle is able to use both of its surfaces for heat dissipation and eliminates the need for insulation It may, however, add appreciably to the weight penalty of the vehicle The type of system to be used will therefore depend on the particular application, and one criterion for the selection of the system would have to be maximum heat rejection for minimum weight

In general, structural and fluid dynamic considerations will dictate the duct-wall thickness and size to insure structural integrity and prevent choking An optimization study would therefore require the determination of the fin thickness (if structurally acceptable) and length that will result in a maximum value of the ratio of heat rejection to system material volume Various analyses, each based on specific conditions, have been presented which treat this problem

Wilkins[1–4] has shown that the profile of a minimum mass radiating fin is proportional to the distance along the fin raised to the $\frac{7}{2}$ power Approximating the fin profile by an optimum triangular shape would add about 10% to the weight of the fin; it would add about 40% if an optimum rectangular shape is used This clearly indicates that, on a weight basis, the rectangular fin is inferior to both the triangular and the absolute optimum tapered fins A great amount of interest, however, centers around the rectangular fin primarily because of its relative simplicity in fabrication Also, a rectangular fin would be the resulting configuration if the skin of the vehicle is used as the radiating surface

Analyses of the fin-tube type configuration using the rectangular fin have been presented by Lieblein,[5] Bartas and Sellers,[6] and Schreiber et al ,[7] thus permitting determination of the optimum fin dimensions when the root temperature is constant Since this would correspond to the case of a coolant condensing system, the optimum fin thickness and length will not vary in the direction of the coolant flow For a noncondensing system, however, the heat loss will be accompanied by a coolant temperature drop, thus resulting in a variable root temperature

The heat-rejection system of the type considered here for a variable root temperature has been treated by Callinan and Berggren[8] on the basis of a fin effectiveness that was assumed constant over the interval of integration in the direction of the coolant flow path As a result of this assumption, the optimum fin dimensions would necessarily be constant along this interval Actually, the optimum fin dimensions would be expected to vary along the tube length because of the variation in the root temperature The manner in which these dimensions and the root temperature vary in such a system has been investigated in the present analysis

Analysis

Consider a space radiator system that is of a fin-tube type configuration, as shown in Fig 1a A hot fluid entering the tube with an initial temperature $T_b(0)$ will dissipate heat by radiation from the surface of the tube and by conduction along the fin and subsequent radiation from the fin surface A balance may therefore be made between the heat lost by the fluid as it flows along the tube and the heat dissipated by radiation The resulting equation is then

$$\frac{m}{2} c_p dT_b = -\sigma\epsilon \left\{ \left[\frac{\pi D}{2} - t_f(y) \right] T^4(0, y) + 2 \int_0^{l(y)} T^4(x\ y)dx \right\} dy \quad (1)$$

where the following assumptions were made: 1) a steady state condition prevails where the heat flow is independent of time; 2) solar and other incident radiation is negligible; 3) the system radiates to an ambient temperature of zero degrees absolute; 4) the surface temperature is constant around the circumference of the tube at any location along the y axis; 5) heat conduction is negligible along the y axis; 6) geometric view factors for fin and tube are equal to 1 0; and 7) the enthalpy change of the fluid is proportional to its temperature change

Equation (1) may be nondimensionalized through the definition of the following transformations:

$$\xi \equiv x/D \qquad \eta \equiv y/D \qquad \xi_l(\eta) \equiv l(y)/D$$
$$\lambda_f(\eta) \equiv t_f(y)/D \qquad \lambda_t \equiv t_t/D$$
$$\theta(\xi, \eta) \equiv T(x, y)/T_b(0) \qquad \theta_b(\eta) \equiv T_b(y)/T_b(0)$$

As a result,

$$\frac{m}{2} c_p d\theta_b = -\sigma\epsilon T_b^3(0)D^2 \left\{ \left[\frac{\pi}{2} - \lambda_f(\eta) \right] \theta^4(0, \eta) + 2 \int_0^{\xi_l(\eta)} \theta^4(\xi, \eta) d\xi \right\} d\eta \quad (2)$$

Furthermore, the fluid temperature θ_b may be eliminated from Eq (2) by relating it to the surface temperature of the tube This is done through a balance of the heat flux through the tube and the radiant heat rate from the tube surface Thus,

$$U[T_b(y) - T(0, y)] = \sigma\epsilon T^4(0, y) \quad (3)$$

where

$$U = 1/[(1/h) + (t_t/k)] \cong h$$

Expressed in nondimensional form, Eq (3) becomes

$$\theta_b(\eta) = \sigma\epsilon T_b^3(0)\theta^4(0, \eta)/U + \theta(0, \eta) \quad (4)$$

which, when substituted into Eq (2), yields

$$[2\beta_1\theta^3(0\ \eta) + 1] \frac{d\theta(0, \eta)}{d\eta} = -\beta_2 \left\{ \left[\frac{\pi}{2} - \lambda_f(\eta) \right] \theta^4(0, \eta) + 2 \int_0^{\xi_l(\eta)} \theta^4(\xi, \eta) d\xi \right\} \quad (5)$$

The parameters β_1 and β_2 in Eq (5) are defined as

$$\beta_1 \equiv 2\sigma\epsilon T_b^3(0)/U \qquad \beta_2 \equiv 2\sigma\epsilon T_b^3(0)D^2/mc_p \quad (5a)$$

As may be seen, Eq (5) represents a relation for the temperature gradient along the tube surface, $d\theta(0, \eta)/d\eta$, and its solution involves the evaluation of the integral

$$\int_0^{\xi_l(\eta)} \theta^4(\xi, \eta) d\xi$$

The value of the integral may be obtained through the use of the conduction equation Thus, for the steady-state condition,

$$d^2T/dx^2 = (2\sigma\epsilon/kt)T^4 \tag{6}$$

or, in terms of the transformed coordinates,

$$d^2\theta/d\xi^2 = [\beta_3/\lambda_f(\eta)]\theta^4(\xi, \eta) \tag{7}$$

where

$$\beta_3 \equiv 2\sigma\epsilon T_b^3(0)D/k \tag{7a}$$

The temperature gradient $d\theta/d\xi$ may now be obtained from Eq (7) as

$$\frac{d\theta}{d\xi} = -\left\{\frac{2}{5}\frac{\beta_3}{\lambda_f(\eta)}[\theta^5(\xi, \eta) - \theta^5(\xi_l, \eta)]\right\}^{1/2} \tag{8}$$

where the constant of integration was evaluated from the boundary conditions at

$$\xi = \xi_l(\eta): \theta(\xi, \eta) = \theta(\xi_l, \eta),\ d\theta/d\xi = 0$$

The integral in Eq (5) therefore becomes

$$2\int_0^{\xi_l(\eta)} \theta^4(\xi, \eta) d\xi = \left\{\frac{8}{5}\frac{\lambda_f(\eta)}{\beta_3}[\theta^5(0, \eta) - \theta^5(\xi_l, \eta)]\right\}^{1/2} = \xi_l(\eta)F \tag{9}$$

with

$$F = F\{\theta\ 0, \eta),\ [\beta_3/\lambda_f(\eta)]^{1/2}\xi_l(\eta)\}$$

Substitution of Eq (9) into Eq (5) finally yields

$$\frac{d\theta(0, \eta)}{d\eta} = -\frac{\beta_2}{2\beta_1\theta^3(0, \eta) + 1}\left\{\left[\frac{\pi}{2} - \lambda_f(\eta)\right]\theta^4(0, \eta) + \xi_l(\eta)F\right\} \tag{10}$$

Equation (10) is the relation used in the present analysis to determine the variation in root temperature along the tube length

The definition of the function F is seen to contain a term for the fin-tip temperature $\theta(\xi_l, \eta)$, which may be related to $\theta(0, \eta)$ and $[\beta_3/\lambda_f(\eta)]^{1/2}\xi_l(\eta)$ through the solution of Eq (8) Thus, from Eq (8),

$$\left[\frac{2}{5}\frac{\beta_3}{\lambda_f(\eta)}\right]^{1/2}\xi_l(\eta) = -\int_{\theta(0\ \eta)}^{\theta(\xi_l\ \eta)} \frac{d\theta}{[\theta^5(\xi, \eta) - \theta^5(\xi_l, \eta)]^{1/2}}$$

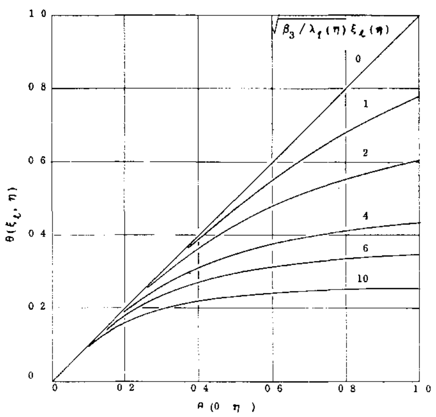

Fig 2 Variation of the tip tempeiatuie with $\theta(0,\eta)$ and $[\beta_3/\lambda_f(\eta)]^{1/2}\xi_l(\eta)$

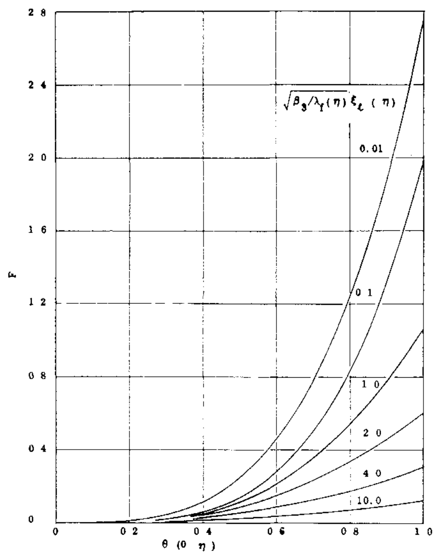

Fig 3 Variation of the parameter F with $\theta(0,\eta)$ and $[\beta_3/\lambda_f(\eta)]^{1/2}\xi_l(\eta)$

or, upon rearrangement,

$$\left[\frac{2}{5}\frac{\beta_3}{\lambda_f(\eta)}\theta^3(\xi_l, \eta)\right]^{1/2}\xi_l(\eta) = \int_1^{\tau_0}\frac{d\tau}{(\tau^5 - 1)^{1/2}} \tag{11}$$

where $\tau \equiv \theta(\xi, \eta)/\theta(\xi_l, \eta)$ and $\tau_0 \equiv \theta(0, \eta)/\theta(\xi_l, \eta)$

Equation (11) is the solution to the one-dimensional steady-state heat-conduction equation [Eq (7)] based on the prescribed boundary conditions, and it involves an integral that has been evaluated in Ref 9 as a function of τ_0 As may further be observed, Eq (11) relates the dimensionless-tip temperature of the fin $\theta(\xi_l, \eta)$ to the root temperature $\theta(0, \eta)$ through the parameter $[\beta_3/\lambda_f(\eta)]^{1/2}\xi_l(\eta)$ This relationship is presented graphically in Fig 2

Equation (11) may also be used to express the function F of Eq (9) in terms of $\theta(0, \eta)$ and $[\beta_3/\lambda_f(\eta)]^{1/2}\xi_l(\eta)$ The variation of F with these parameters is shown in Fig 3

An optimum design criterion in space-radiator systems is the selection of a fin-tube configuration that will produce maximum heat dissipation with minimum system weight The ratio of heat rejection rate to system material volume would therefore be a proper parameter for a system optimiza tion study A relation between such a parameter and the system variables may be obtained as follows: the heat lost by the fluid is

$$dQ = -(m/2)c_pT_b(0)[2\beta_1\theta^3(0, \eta) + 1]d\theta(0, \eta) \tag{12}$$

which, upon definition of a heat-transfer parameter $d\Omega \equiv 2dQ/mc_pT_b(0)$, becomes

$$d\Omega/d\eta = -[2\beta_1\theta^3(0, \eta) + 1][d\theta(0, \eta)/d\eta] \tag{13}$$

Also, the material volume of the differential element $d\eta$ of the fin-tube configuration, assuming a relatively thin tube, may be expressed as

$$\frac{d\psi}{d\eta} \equiv \frac{d}{d\eta}\frac{V}{D^3} = \frac{\pi}{2}\lambda_t + \lambda_f(\eta)\xi_l(\eta) \tag{14}$$

Equations (13) and (14) may now be used for optimizing the radiator configuration Thus, at any value of $\theta(0, \eta)$ corresponding to some particular location η along the tube, optimum values of $\lambda_f(\eta)$ and $\xi_l(\eta)$ will produce a maximum value of $d\Omega/d\psi$ The two conditions that must be satisfied in order to obtain the optimum values of $\lambda_f(\eta)$ and $\xi_l(\eta)$ are then

$$\frac{\partial}{\partial\xi_l}\left(\frac{d\Omega}{d\psi}\right) = 0 \qquad \frac{\partial}{\partial\lambda_f}\left(\frac{d\Omega}{d\psi}\right) = 0$$

Applying these conditions to the foregoing equations yields two relations:

$$4\left[\frac{\pi}{2}\frac{\lambda_t}{\lambda_f(\eta)} + \xi_l(\eta)\right]\theta^5(\xi_l, \eta) - \left\{\left[\frac{\pi}{2} - \lambda_f(\eta)\right]\theta^4(0, \eta) + \xi_l(\eta)F\right\}\left[\frac{3}{2}\frac{\beta_3}{\lambda_f(\eta)}\xi_l^2(\eta)F + 2\theta(0, \eta)\right] = 0 \quad (15)$$

and

$$\left[\frac{\pi}{2}\frac{\lambda_t}{\lambda_f(\eta)} + \xi_l(\eta)\right]\left[2\frac{\lambda_f(\eta)}{\xi_l(\eta)}\theta^4(0, \eta) + \frac{8\lambda_f(\eta)\theta^5(\xi_l, \eta)}{4\lambda_f(\eta)\theta(0, \eta) + 3\beta_3\xi_l^2(\eta)F} - F\right] + 2\left\{\left[\frac{\pi}{2} - \lambda_f(\eta)\right]\times \theta^4(0, \eta) + \xi_l(\eta)F\right\} = 0 \quad (16)$$

Optimum values of $\xi_l(\eta)$ and $\lambda_f(\eta)$ may therefore be obtained for any value of $\theta(0, \eta)$ through a solution of the two simultaneous equations, Eqs (15) and (16) The root temperature $\theta(0, \eta)$ may then be related to the η coordinate along the tube length by means of Eq (10) This, in effect, would provide the variations of the optimum fin dimensions along the coolant flow path

A similar analysis may be performed for a single-surface radiator that uses the skin of the vehicle as the radiating surface Thus, consider the radiator system shown in Fig 1b A heat balance, equivalent to Eq (1) for the double-surface radiator, may be written as

$$\frac{m}{2}c_p dT_b(y) = -\sigma\epsilon\left[\frac{D}{2}T^4(0, y) + \int_0^{l(y)} T^4(x, y)dx\right]dy \quad (17)$$

As a result, the equations for the solution of the problem become, as in the previous analysis,

$$\left[\frac{2}{5}\frac{\beta_3}{\lambda_f'(\eta)}\theta^3(\xi_l, \eta)\right]^{1/2}\xi_l(\eta) = \int_0^{\tau_0}\frac{d\tau}{(\tau^5 - 1)^{1/2}} \quad (18)$$

and

$$\frac{d\theta(0, \eta)}{d\eta} = -\frac{\beta_2}{[2\beta_1\theta^3(0, \eta) + 1]}\left[\frac{1}{2}\theta^4(0, \eta) + \frac{1}{2}\xi_l(\eta)F'\right] \quad (19)$$

where $\lambda_f'(\eta) = 2\lambda_f(\eta)$ and $F' = F\{\theta(0, \eta), [\beta_3/\lambda_f'(\eta)]^{1/2}\xi_l(\eta)\}$ is obtainable from Fig 3 Also,

$$\frac{d\psi}{d\eta} = \frac{\pi}{4}\lambda_t + \left[\frac{1}{2} + \xi_l(\eta)\right]\lambda_f(\eta) \quad (20)$$

and the equation for $d\Omega/d\eta$ is the same as Eq (13)

It now remains to develop the equations that would permit the determination of the optimum fin dimensions Subsequent calculations for the double-surface radiator have indicated that the magnitude of the optimum fin thickness is relatively small and, therefore, may not be structurally feasible Hence, appears that, in general, the fin thickness would be specified as a constant value on the basis of structural integrity and only the fin length would be optimized With that in mind, it would therefore suffice to derive only one equation in terms of optimum $\xi_l(\eta)$ for the single-surface radiator, which would be equivalent to Eq (15) for the double-surface radiator Thus, proceeding in the same manner as was done in obtaining Eq (15), yields the final relation

$$4\left\{\frac{\pi}{2}\frac{\lambda_t}{\lambda_f'} + \left[\frac{1}{2} + \xi_l(\eta)\right]\right\}\theta^5(\xi_l, \eta) - [\theta^4(0, \eta) + \xi_l(\eta)F']\times\left[\frac{3}{2}\frac{\beta_3}{\lambda_f'}\xi_l^2(\eta)F' + 2\theta(0, \eta)\right] = 0 \quad (21)$$

For a given fin thickness and any value of $\theta(0, \eta)$, this equation predicts the value of $\xi_l(\eta)$, which would result in a maximum value of $d\Omega/d\psi$, i e , maximum heat-rejection rate for a minimum weight Thus, a variation may be obtained of $[\xi_l(\eta)]_{\text{opt}}$ with $\theta(0, \eta)$ for a given set of system conditions These two parameters may then be related to the η coordinate through the solution of Eq (19)

Application of Analysis

Consider the space radiator system shown in Fig 1, whose geometric variables are tube diameter and thickness, and fin thickness and length Other variables of the system are thermal conductivity of the structural material, surface emissivity, fluid specific heat, mass rate of flow, and initial temperature

In designing an optimum system, the variables must be selected in such a way as to result in a maximum heat-rejection rate with a minimum structural weight; hence, a maximum value of $d\Omega/d\psi$ in Eqs (13) and (14) or (20) The selection of some of these parameters, however, is limited by structural and fluid dynamic considerations

A minimum value of tube thickness and maximum values of thermal conductivity and surface emissivity would, for example, be desirable for maximizing $d\Omega/d\psi$ The selection of the tube thickness must, however, be based on structural requirements, whereas the latter two parameters will be determined by the type of material, operating temperature range, and, for emissivity, the surface condition

The selection of the tube diameter must be such as to avoid choking of the flow (Mach number <1) at reasonable operating pressures A simple relation between the flow parameters may be obtained from the continuity equation for a perfect gas and one-dimensional flow Thus, from Ref 10,

$$\frac{m}{A}\frac{[T_b(y)]^{1/2}}{P} = \left(\frac{g\gamma\mathfrak{M}}{\mathfrak{R}}\right)^{1/2}\times\frac{M}{\{1 + [(\gamma - 1)/2]M^2\}^{[(\gamma + 1)/2(\gamma - 1)]}}$$

and assuming, for example, an initial Mach number of 0 5 will therefore yield

$$\frac{m}{A}\frac{[T_b(y)]^{1/2}}{P}\frac{1}{\mathfrak{M}^{1/2}} = 265$$

Furthermore, considering the configuration of Fig 1a with 36 lb/hr of hydrogen ($\mathfrak{M} = 2\ 016$) at the initial conditions of $T_b(0) = 2000$°R and $P = 25$ psia will require a tube diameter of 0 467 in ($\sim\frac{1}{2}$ in)

The heat-transfer coefficient inside the tube of the space radiator system may be determined on the basis of fully developed turbulent flow from the relation[11] $Nu = 0\ 023\ Re^{0\ 8}\ Pr^{0\ 4}$ where the fluid properties are normally evaluated at a film temperature that is an average between the fluid and the wall temperature In the present study, however, the wall temperature of the tube is not significantly different from the fluid temperature; hence, the fluid properties will be based on the bulk temperature of the fluid The equation for the heat-transfer coefficient in a circular radiator tube then becomes

$$D^{1\ 8}h/m^{0\ 8} = 2\ 45\ kPr^{0\ 4}/\mu^{0\ 8} = f[T_b(y)]$$

Thus, for hydrogen at 2000°R, the parameter $D^{1\ 8}h/m^{0\ 8} = 5\ 77$ A reduction in the hydrogen temperature to 1500°R, which may correspond to the exit condition, will lower the value of $D^{1\ 8}h/m^{0\ 8}$ to 5 563 The resulting heat-transfer coefficients are then: $T = 2000$° and 1500°R; and $h = 353$ and 340 Btu/ft²-hr °R Since h is not significantly affected by the temperature, it may therefore be taken as constant along the tube at a value of 350 Btu/ft²-hr-°R

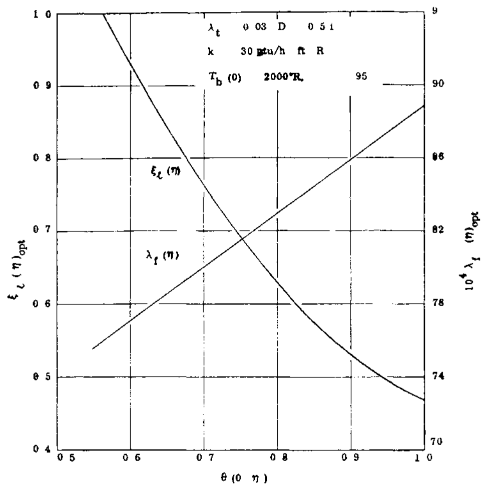

Fig 4 Variation of optimum fin thickness and length with root temperature (double-surface radiator)

The two parameters of the system which may be used to optimize the radiator configuration are the fin thickness and length Specifically, at each point along the tube, there is one value of fin thickness and one corresponding value of fin length that will result in a maximum value of $d\Omega/d\psi$ As was pointed out previously, Eqs (10, 15, and 16) define the optimum values of the fin dimensions and relate them to the η coordinate along the tube Specifically, Eqs (15) and (16) may be solved simultaneously to yield optimum values of $\xi_l(\eta)$ and $\lambda_f(\eta)$ for a range of values of the root temperature $\theta(0, \eta)$ This was done for a set of system conditions that results in a value of $\beta_3 = 0\ 03652$ The results of this calculation, as well as the pertinent system conditions, are shown in Fig 4 As may be observed, the optimum fin length $\xi_l(\eta)$ increases and the optimum fin thickness $\lambda_f(\eta)$ decreases with a decrease in $\theta(0, \eta)$ The variation of $\theta(0, \eta)$ and hence $\xi_l(\eta)$ and $\lambda_f(\eta)$ along the tube may now be obtained with Eq (10) and the results of Fig 4 The resulting profiles of these parameters along the η coordinate are presented in Fig 5

Aside from the shape of the curves, a significant result borne out by Fig 5 is the fact that the magnitude of the optimum fin thickness required for maximizing $d\Omega/d\psi$ is relatively small and, therefore, may not be structurally feasible This would indicate that a space-radiator system of the type considered here would have to be designed on the basis of an optimum fin length that would vary along the tube axis The fin thickness, however, now taken as a constant, would have to be selected on the basis of minimum thickness required to provide structural integrity A configuration using a constant thickness fin would, in addition, be desirable because of its relative simplicity in fabrication

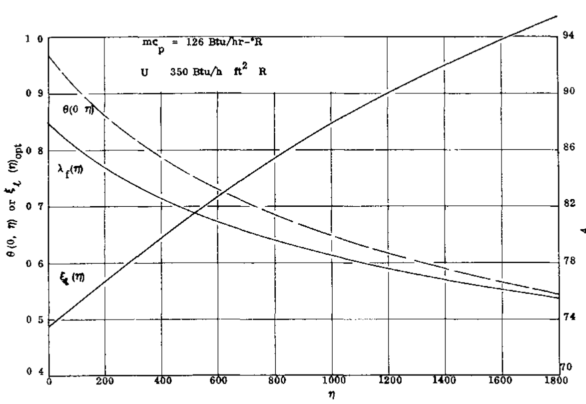

Fig 5 Variation of optimum fin thickness and length and root temperature along the tube (double-surface radiator)

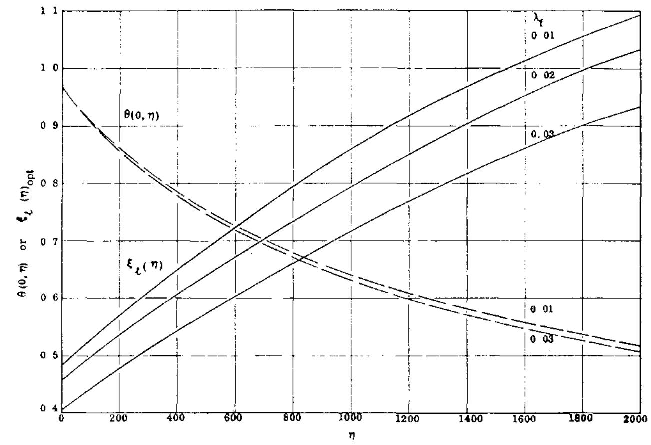

Fig 6 Variation of optimum fin length and root temperature along the tube for fixed values of fin thickness (double-surface radiator)

Only Eqs (10) and (15) are therefore required for an optimization study because the value of λ_f is now specified Thus, Eq (15) may be solved to yield the variation of $\xi_l(\eta)_{opt}$ with $\theta(0, \eta)$ for several values of λ_f These parameters are then related to η through Eq (10) in the same manner as was done previously for Fig 5 The results of such a study using three values of λ_f are shown in Fig 6 As may be seen for the range of fin thicknesses considered, an increase in λ_f is accompanied by a decrease in the optimum fin length $\xi_l(\eta)$ and only a relatively slight change in $\theta(0, \eta)$

In the preceding study, representative profiles have been obtained for the fin dimensions that would result in an optimum heat-rejection system Since a variable-length fin may not always be practical, the question therefore arises as to the weight penalty that would result when an equivalent constant-length fin is used A comparison was therefore made between the method of Ref 8 (which treats an optimum constant-length fin for a variable root temperature) and the present analysis for the variable-length fin, using the single-surface radiator configuration of Fig 1b

The results of the comparison as well as the values of the specified system parameters are presented in Fig 7 As may be observed from the figure, the optimum constant-length fin of Ref 8 represents approximately a mean value of the fin profile of the present analysis Also, the root tempera-

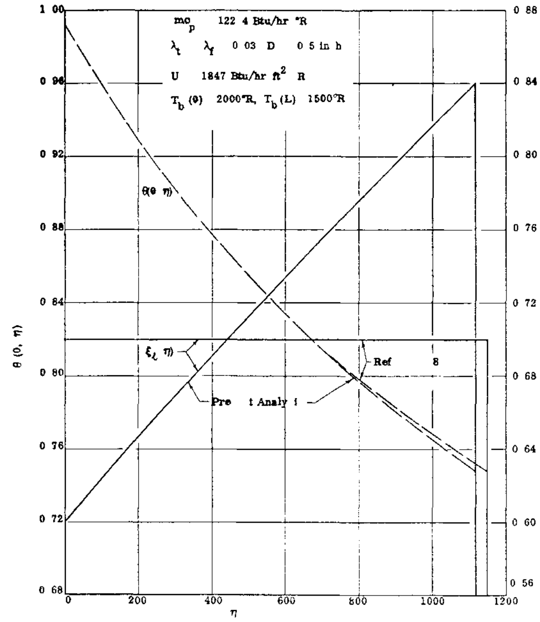

Fig 7 Comparison of the present analysis with the method of Ref 8 for the single-surface radiator

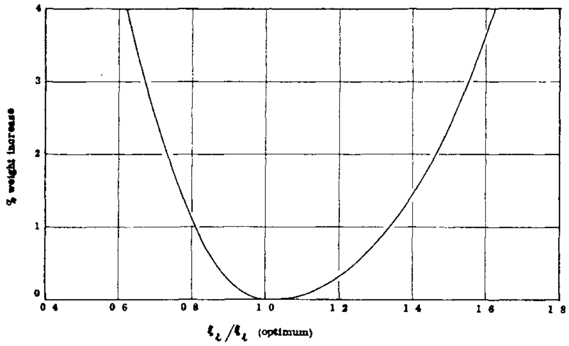

Fig 8 Effect of off-optimum constant length fin design on system weight for the single-surface radiator

ture profile $\theta(0, \eta)$ is seen to be the same for both fins for a considerable length of tube (to $\eta \simeq 700$) and only a relatively moderate deviation is observed in the magnitude of $\theta(0, \eta)$ for the two fins beyond $\eta = 700$ The result of this deviation, however, is sufficient to require additional length in η for the constant-length fin in order to attain the specified fluid exit temperature This is indicated in Fig 7 by the two vertical lines that represent the locations along the η coordinate where the fluid exit temperature reaches the value $\theta(0, \eta) = 0\ 748$ (1500°R)

The comparison in Fig 7 indicates that, for the space-radiator system considered, a 1 5% weight penalty would be added to the system if the optimum fin profile of the present analysis is replaced by the constant-length optimum fin of Ref 8 This represents a rather small weight increase, and it would therefore tend to justify the use of a constant-length fin which may be easier to fabricate

In general, however, the efficiency of a space-radiator system (and, hence, its weight for given inlet and exit fluid conditions) is not significantly affected by moderate deviations in the fin length from its optimum value This may be observed in Fig 8, where the optimum constant-length fin of Fig 7 (based on the method of Ref 8) was used as the basis for comparison As shown in Fig 8, an increase in fin length from its optimum value by 60% would be accompanied by a weight penalty of only 3 65% and a decrease in fin length by 30% would result in a system weight increase of merely 2 5% This indicates that large deviations from optimum design condition would introduce relatively small weight penalties and, therefore, the 1 5% difference between the present method and that of Ref 8 may, in fact, represent a large deviation from the optimum

Conclusions

A method has been developed for obtaining profiles of the optimum fin dimensions for the configurations of Fig 1 Calculations based on this method indicate that the optimum fin length increases and the optimum fin thickness decreases along the fluid flow path

Aside from the mode of variation of the foregoing parameters, a significant result borne out by these calculations is the fact that the magnitude of the optimum fin thickness is, for the representative case considered, too small to be structurally feasible This indicates that the fin thickness, now taken as a constant, would be selected on the basis of minimum thickness required to provide structural integrity and hence, only the fin length would be optimized

A comparison was made between the method of Ref 8, which treats an optimum constant-length fin for a variable root temperature, and the present analysis for the variable-length fin The results indicate that, for the configuration and system conditions used, the radiator system with the constant-length fin is 1 5% heavier than the system that uses the fin profile of the present method This represents a rather small weight increase, and it would therefore tend to justify the use of a constant-length fin which may be easier to fabricate It was shown, however, that large deviations from the optimum design condition would introduce relatively small weight penalties, and therefore the 1 5% difference between the present method and that of Ref 8 may, in fact, represent a large deviation from the optimum Hence, it may be concluded that, for the configurations of Fig 1 and for given inlet and exit fluid temperatures, the present analysis yields the lightest radiator system However, strict adherence to optimum design conditions is not generally necessary

In conclusion, it should be noted that the present theory has omitted the radiation exchange between the base and the fin For a single-surface radiator this effect does not appear, since the tube is attached to the nonradiating side of the vehicle's skin Obviously, for the double-surface radiator system, the base and fin radiation exchange does influence the heat-rejection rate As shown in Ref 8 this effect amounts to approximately 8% of the heat-rejection rate for a typical case From the analysis appearing in Ref 12, a significant share of the total heat transfer is dissipated by the base sur face Since the base-surface heat loss is near 90% of the radiant energy emission, the total heat dissipation is considerably less sensitive to the effect of the radiant interaction between the base and fin than to the heat dissipated by the fin along Hence, the analysis in the present paper does not incorporate the second-order effect of the base and fin radiation exchange

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